DIMENSION REDUCTION AND SHRINKAGE

Part I. Variable Selection and Ridge Regression

1. VARIABLE SELECTION

```
> attach(Auto)
> library(leaps)
> reg.fit = regsubsets( mpg ~ cylinders + displacement + horsepower + weight + acceleration + year, Auto )
> summary(reg.fit)
Selection Algorithm: exhaustive
         cylinders displacement horsepower weight acceleration year
1  ( 1 ) " "       " "          " "        "*"    " "          " " 
2  ( 1 ) " "       " "          " "        "*"    " "          "*" 
3  ( 1 ) " "       " "          " "        "*"    "*"          "*" 
4  ( 1 ) " "       "*"          " "        "*"    "*"          "*" 
5  ( 1 ) "*"       "*"          " "        "*"    "*"          "*" 
6  ( 1 ) "*"       "*"          "*"        "*"    "*"          "*" 
```

 # This command finds the best model for each p = number of independent variables. The best model is determined by the lowest RSS.

 # Next, choose the best p according to some criteria:

```
> summary(reg.fit)$adjr2                              # Adjusted R^2
[1] 0.6918423 0.8071941 0.8071393 0.8067872 0.8067841 0.8062826
> summary(reg.fit)$cp                                 # Mallows Cp
[1] 232.396144   1.169751   2.284200   3.992019   5.000800   7.000000
> summary(reg.fit)$bic                                # BIC = Bayesian information criterion
[1] -450.5016 -629.3564 -624.2828 -618.6081 -613.6448 -607.6743
```

 # Recall that plain R^2 is not a fair measure of performance. It always increases with p:

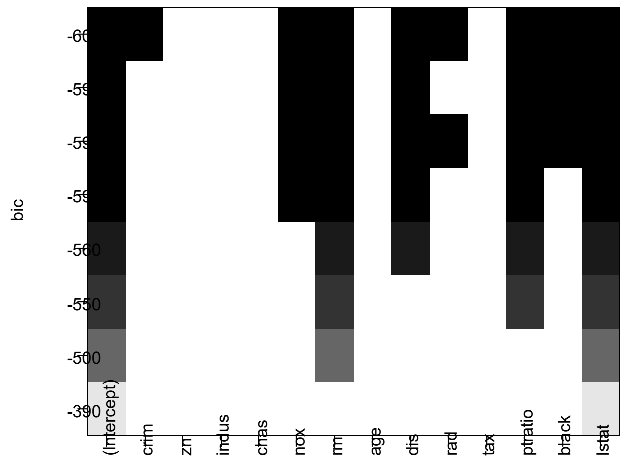
```
> summary(reg.fit)$rsq
[1] 0.6926304 0.8081803 0.8086190 0.8087638 0.8092549 0.8092553
```

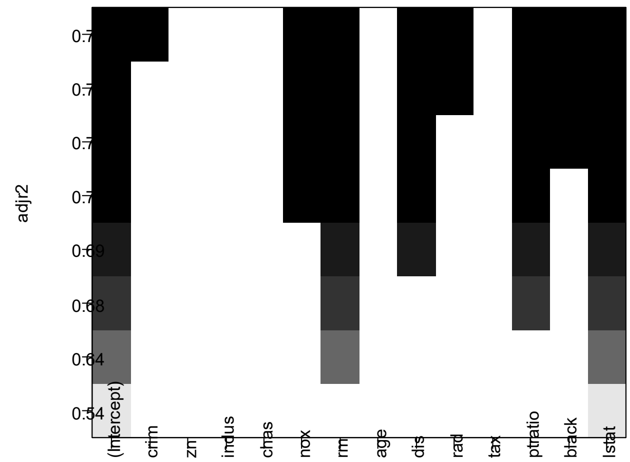
 # For stepwise or backward elimination variable selection, use method="forward" or method="backward".

```
> library(MASS)
> reg = regsubsets( medv ~ ., data=Boston, method = "backward" )
```

 # There is a nice way to visualize results, ranking models by the chosen "scale". Black color means the variable is included into the model, white means it is excluded.

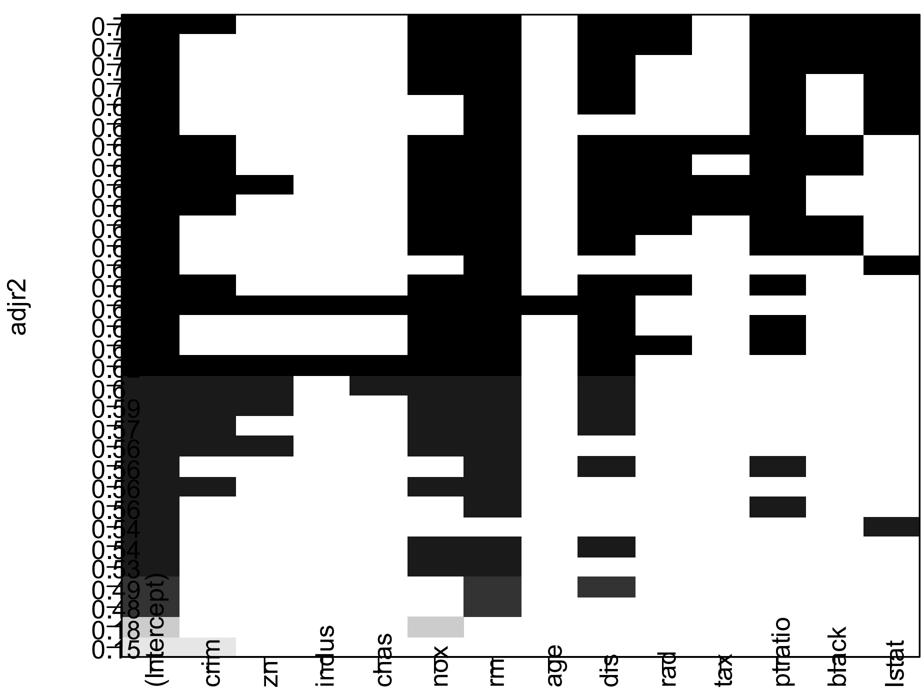
```
> plot(reg)
> plot(reg, scale = "adjr2" )
```





 # To see more models, use option "nbest", which is the number of models of each size p to be compared.

```
> reg = regsubsets( medv ~ ., data=Boston, method = "backward", nbest=4 )
> plot(reg, scale = "adjr2" )
```



 # We can also choose the best model by means of a stepwise procedure, starting with one model and ending with another.

```
> null = lm( medv ~ 1, data=Boston )
> full = lm( medv ~ ., data=Boston )
> step( null, scope=list(lower=null, upper=full), direction="forward" )
Start:  AIC=2246.51
medv ~ 1
```

```
            Df Sum of Sq   RSS    AIC
+ lstat    1   23243.9 19472 1851.0          # Compare contributions of
+ rm       1   20654.4 22062 1914.2          # remaining independent variables
+ ptratio  1   11014.3 31702 2097.6
+ indus    1    9995.2 32721 2113.6
+ tax      1    9377.3 33339 2123.1
+ nox      1    7800.1 34916 2146.5
+ crim     1    6440.8 36276 2165.8
+ rad      1    6221.1 36495 2168.9
+ age      1    6069.8 36647 2171.0
+ zn       1    5549.7 37167 2178.1
+ black    1    4749.9 37966 2188.9
+ dis      1    2668.2 40048 2215.9
+ chas     1    1312.1 41404 2232.7
<none>                 42716 2246.5

Step:  AIC=1851.01
medv ~ lstat

            Df Sum of Sq   RSS    AIC
+ rm       1    4033.1 15439 1735.6
+ ptratio  1    2670.1 16802 1778.4
+ chas     1     786.3 18686 1832.2
+ dis      1     772.4 18700 1832.5
+ age      1     304.3 19168 1845.0
+ tax      1     274.4 19198 1845.8
+ black    1     198.3 19274 1847.8
+ zn       1     160.3 19312 1848.8
+ crim     1     146.9 19325 1849.2
+ indus    1      98.7 19374 1850.4
<none>                 19472 1851.0
+ rad      1      25.1 19447 1852.4
+ nox      1       4.8 19468 1852.9

... < truncated > ...

Step:  AIC=1585.76
medv ~ lstat + rm + ptratio + dis + nox + chas + black + zn +
    crim + rad + tax

          Df Sum of Sq   RSS    AIC
<none>                 11081 1585.8
+ indus   1   2.51754 11079 1587.7
+ age     1   0.06271 11081 1587.8

Call:
lm(formula = medv ~ lstat + rm + ptratio + dis + nox + chas +
    black + zn + crim + rad + tax, data = Boston)

Coefficients:
(Intercept)        lstat           rm      ptratio          dis          nox
  36.341145    -0.522553     3.801579    -0.946525    -1.492711   -17.376023
       chas        black           zn         crim          rad          tax
   2.718716     0.009291     0.045845    -0.108413     0.299608    -0.011778
```

 # The final model contains variables lstat, rm, ptratio, dis, nox, chas, black, zn, crim, rad, and tax.

---

2. RIDGE REGRESSION

 # Dataset "Boston" has some strong correlations, resulting in multicollinearity.

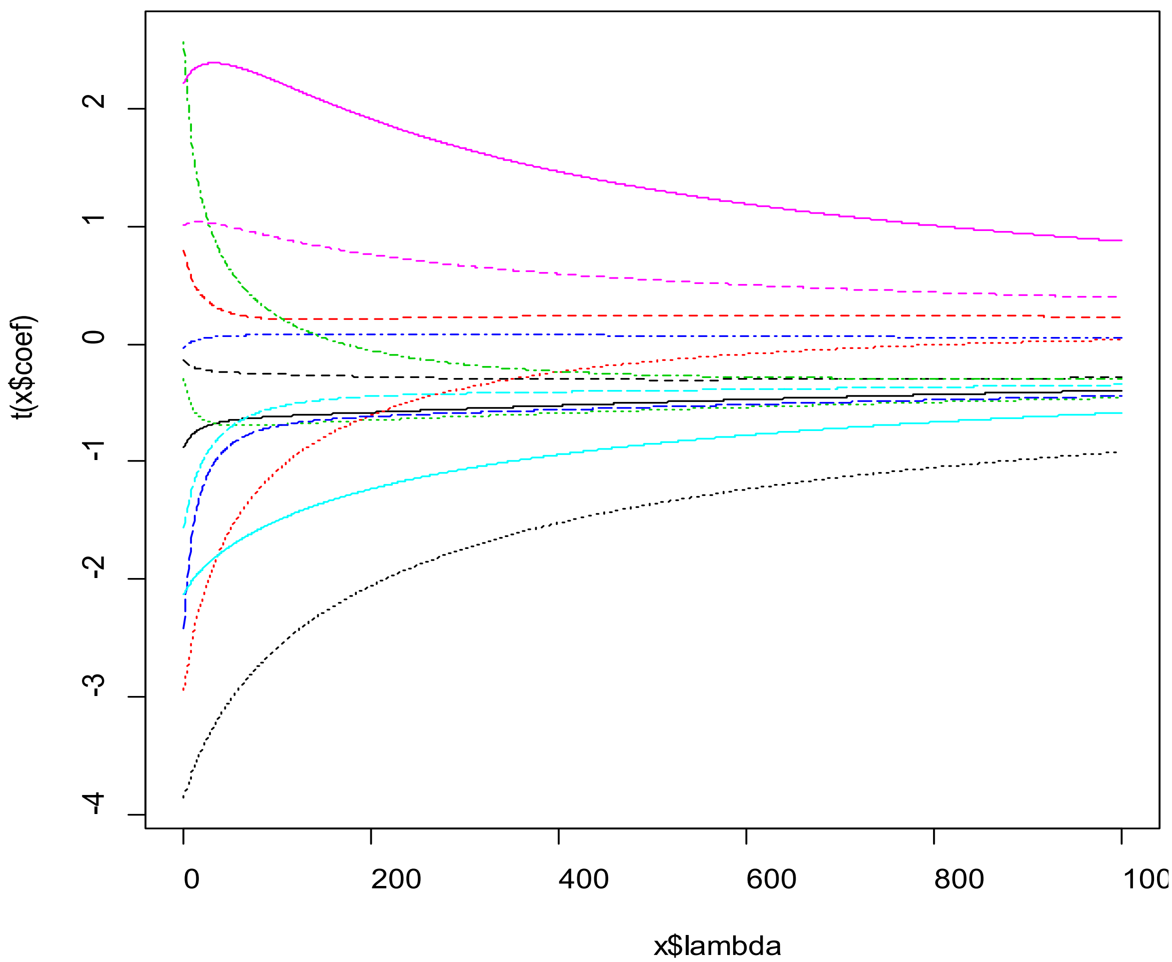
```
> cor(Boston)
```

 # Apply ridge regression

```
> lm.ridge( medv~., data=Boston, lambda=0.5 )
                      crim            zn         indus          chas
 36.270091954 -0.107524388   0.046092635   0.018815242   2.693518778
          nox           rm           age           dis           rad
-17.646013524   3.816019080  0.000578687  -1.469191794   0.301928336
          tax      ptratio         black         lstat
 -0.012134702 -0.950831885   0.009309388  -0.523845421
```

 # We can see how the slopes change with penalty lambda. These are estimated slopes for different lambda. When lambda=0, we get LSE. Large lambda forces them toward 0.

```
> rr = lm.ridge(medv ~ ., data=train, lambda=seq(0,1000,1) )
> rr
> plot(rr)
```



 # To choose a good lambda, fit ridge regression with various lambda and compare prediction performance.

```
> select(rr)
modified HKB estimator is 4.321483
modified L-W estimator is 3.682107
smallest value of GCV  at 5
```

 # So, the best lambda is around 5. We'll look closer at that range.

```
> rr = lm.ridge( medv~., data=Boston, lambda=seq(0,10,0.01) )
```

```
> select(rr)
modified HKB estimator is 4.594163
modified L-W estimator is 3.961575
smallest value of GCV  at 4.26
> plot(rr$lambda,rr$GCV)
```